Optimizing at All Scales: Edge (Non)linear Model Predictive Control from MCUs to GPUs

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***Abstract*—Model predictive control (MPC) is a powerful tool for controlling highly dynamic robotic systems subject to complex constraints. However, MPC, and its underlying (nonlinear) optimization algorithms, are often too computationally demanding to meet real-time rates for robotic platforms, both large and small. These problems are exacerbated by the end of Dennard Scaling and Moore's law, which have led to a utilization wall that limits the performance a single CPU chip can deliver. As such, we now need to look to the field of software performance engineering to co-design our solvers for their target hardware architectures. As such, in our recent works, by leveraging a combination of parallelism, approximation, and structure exploitation, we have enabled and accelerated (nonlinear) trajectory optimization solvers for real-time performance on non-standard computational hardware, ranging from microcontrollers (MCUs) to graphical processing units (GPUs). This has led to real-time MPC onboard an MCU powered 27g quadrotor for dynamic obstacle avoidance, as well as simulated whole-body nonlinear MPC at kHz rates for a GPU powered manipulator for high speed trajectory tracking.**

I. Introduction

Model Predictive Control (MPC) has enabled reactive and dynamic online control for robots while respecting complex control and state constraints such as those encountered during dynamic obstacle avoidance and contact events [56, 10, 28, 21, 17, 50, 53]. However, despite MPC's many successes, its practical application is often hindered by computational limitations, which can necessitate algorithmic simplifications, especially for systems requiring high control rates for safe and effective operation [59, 42, 33, 34].

Compounding this issue, the end of Moore's Law and Dennard Scaling have led to a utilization wall that limits the performance a single CPU chip can deliver [55, 12]. As such, for computationally bounded algorithms, like MPC, computer scientists have had to look beyond the CPU to exploit large-scale parallelism available on alternative computing platforms such as GPUs. Several recent efforts have shown that significant computational benefits are possible by exploiting the natural parallelism in the computation of dynamics and cost functions on GPUs and FPGAs [4, 40, 44, 33, 45, 24, 35, 58]. However, multiple-shooting and consensus approaches to computing trajectory updates at each algorithmic iteration [15, 14, 20, 18, 29, 26] have only seen modest gains when implemented on alternative hardware platforms [42, 43].

At the same time, there has been an explosion of interest in tiny, low-cost robots that can operate in confined spaces, making them a promising solution for applications ranging from emergency search and rescue [30] to routine monitoring and maintenance of infrastructure and equipment [9, 11]. These robots are limited to low-power, resource-constrained microcontrollers (MCUs) for their computation [13, 41]. These microcontrollers feature orders of magnitude less RAM, flash memory, and processor speed compared to the CPUs and GPUs. Consequently, many examples in the literature of intelligent robot behaviors executed on these tiny platforms rely on off-board computers [2, 54, 22, 27, 57, 52, 8].

In this work we show that in order to overcome these challenges at all scales we need to look to the field of software performance engineering and leverage holistic algorithm-hardware co-design. Through this approach we can meet performance targets while respecting real-world computing constraints. We demonstrate the power of this approach through two projects. First we describe MPCGPU (https://github.com/a2r-lab/MPCGPU) [1, 7], a GPU-accelerated, NMPC solver that exploits the structured sparsity and the natural parallelism in direct trajectory optimization through a custom preconditioned conjugate gradient solver at its core. Our experiments show that MPCGPU increases the scalability and real-time performance of NMPC, solving larger problems, at faster rates. Next we describe TinyMPC (https://tinympc.org) [36, 48], an MCU-optimized implementation of convex MPC using the alternating direction method of multipliers (ADMM) algorithm. To the best of the authors' knowledge, TinyMPC is the first MPC solver tailored for execution on these MCUs that has been demonstrated onboard a highly dynamic, compute-limited robotic system, and can support second-order cone constraints.

II. Background

A. Direct Trajectory Optimization

In most MPC formulations, a trajectory optimization problem [5] is solved at each control step. These problems solve a (nonlinear) optimization problem to compute a robot's path through an environment as a series of states $X = \{x_0, \cdots, x_N\}$ and controls $U = \{u_0, \cdots, u_{N-1}\}$ for $x \in \mathbb{R}^n$ and $u \in \mathbb{R}^m$, model the robot as a discrete-time dynamical system,

$$x_{k+1} = f(x_k, u_k, h), \quad x_0 = x_s, \tag{1}$$

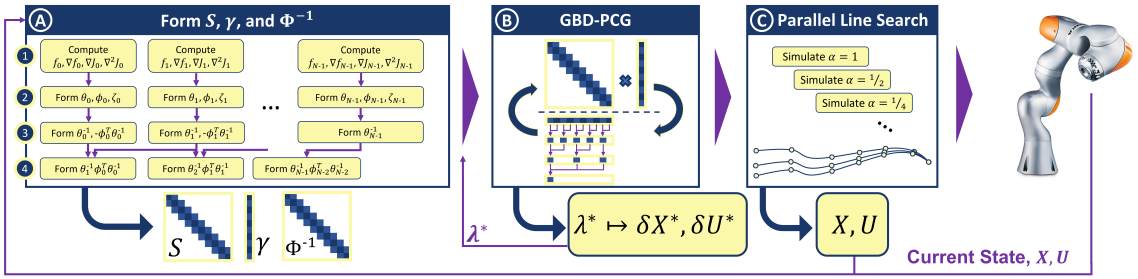


Fig. 1: A high level overview of MPCGPU which: 1) in parallel on the GPU computes S, γ, and Φ^{-1} and stores those values in an optimized dense format, 2) uses our GBD-PCG solver to compute λ^* and reconstructs $\delta X^*, \delta U^*$ through GPU-friendly matrix-vector multiplications and vector reductions, and 3) leverages a parallel line search to compute the final trajectory, X, U. This trajectory is then passed to the (simulated) robot and the current state of the (simulated) robot is measured and fed back into our solver which is run again, warm-started with our last solution.

with a timestep h, minimize an additive cost function,

$$J(X,U) = \ell_f(x_N) + \sum_{k=0}^{N-1} \ell(x_k, u_k), \tag{2}$$

and may also be subject to additional constraints (e.g., torque limits, obstacle avoidance constraints),

$$h(X,U) \leq 0. \tag{3}$$

In the nonlinear case, Taylor approximations of $f(\cdot)$, $h(\cdot)$, and $J(\cdot)$ are taken at each iteration of the algorithm, and regularization is applied, to produce a convex optimization problem. The solution to this convex problem is then often followed by a line search [37].

B. The Alternating Direction Method of Multipliers

The alternating direction method of multipliers (ADMM) is a popular and efficient approach for solving convex optimization problems [6]. Given a generic problem of minimizing a convex function $f(x)$ according to a constraint $x \in \mathcal{C}$, we can form the equivalent problem (with the slack z),

$$\begin{aligned} \min_{x} \quad & f(x) + I_{\mathcal{C}}(z) \\ \text{subject to} \quad & x = z. \end{aligned} \tag{4}$$

The augmented Lagrangian of the transformed problem (4) is (with Lagrange multiplier λ and scalar penalty weight ρ):

$$\mathcal{L}_A(x,z,\lambda) = f(x) + I_{\mathcal{C}}(z) + \lambda^\intercal(x-z) + \frac{\rho}{2}||x-z||_2^2. \tag{5}$$

Thus, if we alternate minimization over x and z, we arrive at the three-step ADMM iteration,

$$\text{primal update} : x^+ = \arg\min_x \mathcal{L}_A(x,z,\lambda), \tag{6}$$

$$\text{slack update} : z^+ = \arg\min_z \mathcal{L}_A(x^+,z,\lambda), \tag{7}$$

$$\text{dual update} : \lambda^+ = \lambda + \rho(x^+ - z^+), \tag{8}$$

where the last step is a gradient-ascent update on the Lagrange multiplier [6]. These steps can be iterated until a desired convergence tolerance is achieved.

In the special cases of quadratic programs (QP) or a second-order cone programs (SOCP), each step of the ADMM algorithm becomes very simple to compute: the primal update is the solution to a linear system, and the slack update is a linear or conic projection. ADMM-based QP and SOCP solvers such as OSQP [51] and SCS [39] are state-of-the-art.

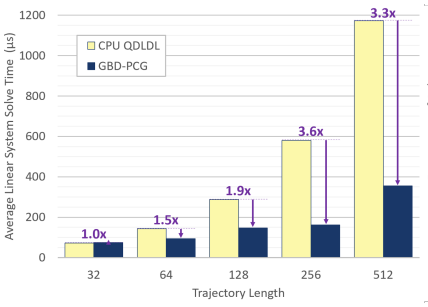
C. Iterative Linear System Solvers

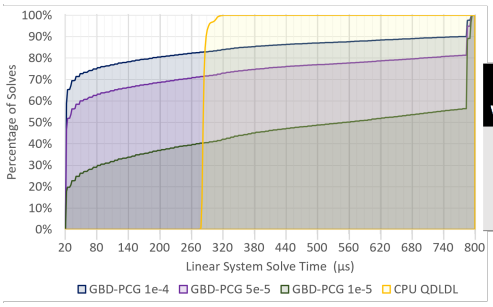
While solvers like OSQP leverage factorization-based approaches to solving their underlying linear systems (in particular the state-of-the-art QDLDL solver for OSQP), iterative methods solve the problem $Ax = b$ for a given A and b by iteratively refining an estimate for x up to some tolerance ϵ. The most popular of these methods is the conjugate gradient (CG) algorithm which has been used for state-of-the-art results on large-scale optimziation problems on the GPU [32, 49]. The convergence rate of CG is directly related to the spread of the eigenvalues of A [38]. Thus, a preconditoning matrix $\Phi \approx A$ is often applied to instead solve the equivalent problem with better numerical properties: $\Phi^{-1}Ax = \Phi^{-1}b$. To do so, the preconditioned conjugate gradient (PCG) algorithm leverages matrix-vector products with A and Φ^{-1}, as well as vector reductions, both parallel friendly operations.

III. MPCGPU

A. Design and Implementation

As shown in Figure 1, our approach is broken down into the three step process found in most direct methods [37], but optimized to expose GPU-friendly computational patterns. First, at each control step we construct a Taylor expansion of the original problem, forming a QP. To form a symmetric positive (semi-)definite linear system that we can solve with PCG, we leverage the Schur complement reformulation of the resulting KKT system. We form each block row of the Schur complement system, S and γ, as well as our preconditioner, Φ^{-1}, in parallel, by taking advantage of the structured sparsity of those matrices. To ensure efficient computation of the underlying dynamics and kinematic quantities, we leverage the GRiD library [45]. Next, we use our custom GPU-optimized, warm-started, block-tridiagonal PCG solver, GBD-PCG, to compute the optimal Lagrange multipliers, λ^*, and reconstruct the optimal trajectory update, $\delta X^*, \delta U^*$. Finally we leverage a parallel line search to compute the final trajectory X, U, which not only reduces latency of the update step, but can also





MPCGPU with GBD-PCG		Knot Points 32	64	128	256	512
Control Rate	250Hz	22.2	19.7	15.4	5.2	4.4
	500Hz	10.3	10.6	8.0	4.6	3.0
	1kHz	4.9	5.2	3.7	2.4	1.7

Fig. 2: (Left) Average linear system solve time ($N = 32 \dots 512$). (Center) Linear system solve cumulative distribution function ($N = 128$). (Right) Average number of trajectory optimization iterations of MPCGPU at each control step. Results were collected on high-performance workstation with a 3.2GHz 16-core Intel i9-12900K and a 2.2GHz NVIDIA GeForce RTX 4090 GPU which ran 100 NMPC trials of end-effector position tracking for a simulated Kuka IIWA-14 executing a 10 second, 5 goal, pick-and-place circuit, with thousands of underlying linear system solves.

improve the convergence of NMPC [42]. We send this to the (simulated) robot for execution and simultaneously measure the current state for our next control step.

B. Experiments and Results

As shown in Figure 2 (left), GBD-PCG outperforms the state-of-the-art CPU-based QDLDL solver [51] across most problem sizes, obtaining as much as a 3.6x average speedup.

In most cases, the speedup is much larger than this, as iterative methods can exit early when warm-started. Using the 128 knot point problem as a case study, as shown in Figure 2 (center), for $\epsilon = 1e^{-4}$, 65% of GBD-PCG solves are $\geq$10x faster than the fastest QDLDL solve, and the slowest GBD-PCG solve is only 2.5x slower than the slowest QDLDL solve (with only 10% $\geq$2x slower).

Figure 2 (right) shows the number of average trajectory optimization iterations MPCGPU can achieve while meeting the specified control rates and trajectory lengths. Our GPU-first approach enables us to solve 512 knot point trajectories at 1kHz and execute 8 iterations for 128 knot points at 500Hz, for a per-iteration rate of 4kHz. This compares favorably to previously reported results of 500hz to 1kHz per-iteration rates for trajectories of 30 to 120 knot points using state-of-the-art CPU-based [29, 19] and GPU-based [42] solvers.

IV. TinyMPC

A. Design and Implementation

TinyMPC trades generality for speed and low-memory utilization to enable real-time use on MCUs by exploiting the structure of the MPC problem. Specifically, we leverage the closed-form Riccati solution to the LQR problem to compute the primal update in (6), accounting for the standard dynamics constraints, and leave any additional constraints to be handled by the remainder of the ADMM algorithm.

In particular, given a long enough horizon, the Riccati recursion (9) converges to the constant solution of the infinite-horizon LQR problem [25]. Thus, we pre-compute a single LQR gain matrix K_∞ and cost-to-go Hessian P_∞. Then, instead of solving the full LQR update at each timestep:

$$\begin{aligned} K_k &= (R + B^\intercal P_{k+1} B)^{-1}(B^\intercal P_{k+1} A) \\ d_k &= (R + B^\intercal P_{k+1} B)^{-1}(B^\intercal p_{k+1} + r_k) \\ P_k &= Q + K_k^\intercal R K_k + (A - BK_k)^\intercal P_{k+1}(A - BK_k) \\ p_k &= q_k + (A - BK_k)^\intercal (p_{k+1} - P_{k+1} B d_k) + K_k^\intercal (R d_k - r_k). \end{aligned} \tag{9}$$

We instead cache the following matrices from (9):

$$\begin{aligned} C_1 &= (R + B^\intercal P_\infty B)^{-1}, \\ C_2 &= (A - BK_\infty)^\intercal, \end{aligned} \tag{10}$$

and then only need to update the linear terms during each ADMM iteration:

$$\begin{aligned} d_k &= C_1(B^\intercal p_{k+1} + r_k), \\ p_k &= q_k + C_2 p_{k+1} - K_\infty^\intercal r_k. \end{aligned} \tag{11}$$

As a result, we avoid online matrix factorization and only compute matrix-vector products. We also dramatically reduce memory footprint by only storing a few vectors per time step.

We also note that the slack update in (7) can be written as the operator Π that projects the slack variable onto the feasible space. For linear inequality constraints, the projection is onto a set of bounds defined by the element-wise operator

$$\Pi(z) = \max(z_l, \min(z_u, z)), \tag{12}$$

where z corresponds to the state and control input slack variables. As the structure of the ADMM algorithm inherently isolates the projection step, we can also replace the projection operator in the slack update (12) with the SOC projection:

$$\Pi_{\mathcal{K}}(z) = \begin{cases} 0, & \|v\|_2 \leq -a, \\ z, & \|v\|_2 \leq a, \\ \frac{1}{2}\left(1 + \frac{a}{\|v\|_2}\right) \begin{bmatrix} v \\ \|v\|_2 \end{bmatrix}, & \|v\|_2 > |a|, \end{cases} \tag{13}$$

where $v = [z_1, \dots, z_{n-1}]^\intercal$, $a = z_n$. Here, $z_i, i = 1, ..., n$ is any vector subset of the state or control slack variables.

B. Experiments and Results

We compare TinyMPC against state-of-the-art solvers for two problems while varying the number of states and the horizon length. The first is a predictive safety filtering problem with box constraints on the state and input. The second is

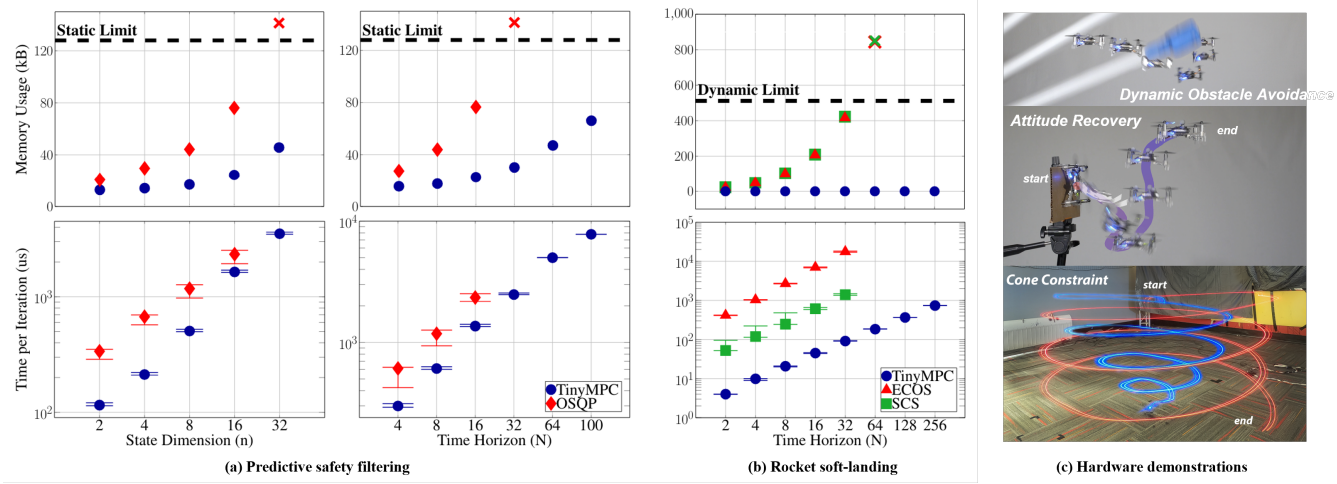


Fig. 3: (a) Compares memory usage (top) and average iteration times (bottom) for TinyMPC and OSQP on a QP-based predictive safety filtering task using a 168 MHz STM32F405 with 1 MB of Flash, and 128 kB of RAM. In the first column, the time horizon was kept constant at $N = 10$ while the state dimension n ranged from 2 to 32 and the input dimension was set to half of the state dimension. In the second column, the state and control input were held constant at $n = 10$ and $m = 5$ while N ranged from 4 to 100. (b) Compares average iteration times (top) and memory usage (bottom) for TinyMPC and ECOS and SCS on an SOCP-based rocket soft-landing using a 600 MHz Teensy 4.1 with 7.75 MB of flash, and 512 kB of tightly coupled RAM. In this experiment $n = 6$ and $m = 3$ while N varied from 2 to 256. The error bars represent the maximum and minimum time taken per iteration for all MPC steps performed for a specific problem. The black dotted lines denote memory thresholds. Across all problems, TinyMPC requires the least memory and is the fastest. (c) Shows the results of hardware experiments for real-time dynamic obstacle avoidance (top), recovery from a 90° attitude error (middle), and tracking a descending helical reference (red) with its position subject to a 45° second-order cone glideslope (blue).

a rocket soft-landing problem with a second-order cone constraint on the thrust vector. The safety filter QP is benchmarked against OSQP and the rocket soft-landing SOCP is benchmarked against ECOS and SCS. The microcontroller results are reported in Figure 3 (a) and (b). Across both problems we find that TinyMPC outperforms comparable state-of-the-art solvers both in terms of latency and in terms of memory usage. In fact, alternative solvers often exceed the memory limits available on our MCU hardware. For example, OSQP exceeds memory limits and cannot be run onboard the STM32F405 for $n, N \geq 32$, and ECOS and SCS exceed the limits of the higher resourced Teensy 4.1 at $N \geq 64$, while TinyMPC scales to $N, n = 100$ and $N = 256$ respectively.

Figure 3 (c) shows the results of hardware demonstrations showing real-time dynamic obstacle avoidance (top), recovery from a 90° attitude error (middle), and tracking a descending helical reference with its position subject to a 45° second-order cone glideslope (bottom), demonstrating the real-world, real-time applicability of TinyMPC.

V. Conclusion and Future Work

Throughout these works we show that in order to overcome computational challenges in robotics at all scales we need to look to the field of software performance engineering and leverage holistic algorithm-hardware co-design. With MPCGPU (https://github.com/a2r-lab/MPCGPU) [1, 7], we show that by leveraging structure sparsity and natural parallelism we can develop a faster-than-state-of-the-art nonlinear MPC solver on the GPU. With TinyMPC (https://tinympc.org) [36, 48], we show that through principled algorithmic simplifications, convex MPC can be run in real-time on a MCU, enabling highly dynamic, compute-limited robotic systems to perform real-time obstacle avoidance and support second-order cone constraints. We hope this work increases interest in co-design, performance engineering, and computational systems for robotics.

In future work we hope to leverage these performant solutions to begin to bridge the gap between optimal control and learning-based control and integrate both control stacks with additional onboard sensors to enable fully autonomous operation on real-world tasks. We are particularly interested in leveraging MPCGPU to support faster actor-critic approaches to MPC [16, 3, 31, 47, 46], and to add adaptive control [23], parameter learning [60], and model-based RL [22] approaches to TinyMPC, enabling it to adjust to a changing real-world environments despite caching certain computations offline.

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