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Robust Optimal Control for Nonlinear Systems Based on Adaptive Reinforcement Learning

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ONE LOVE. ONE FUTURE.

Contents

Tracking control

- Nonlinear systems with disturbances and uncertainties
- Robust adaptive approach
- Traditional optimal approach

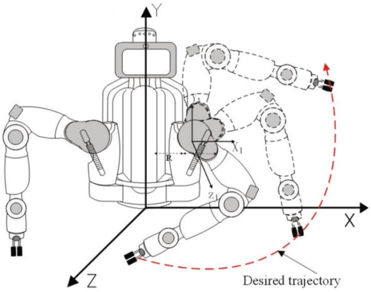


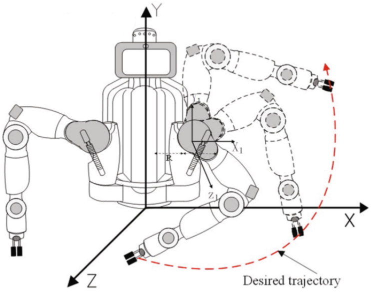
Figure 1: Tracking control

Tracking control

- Nonlinear systems with disturbances and uncertainties
- Robust adaptive approach
- Traditional optimal approach

Reinforcement learning (RL)

- Multi-purpose
- Multi-component



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Tracking control

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Objectives

- Deal with disturbances/uncertainties
- Improve tracking performance
- Adaptive optimal solution

⇒ **Propose two control structures**

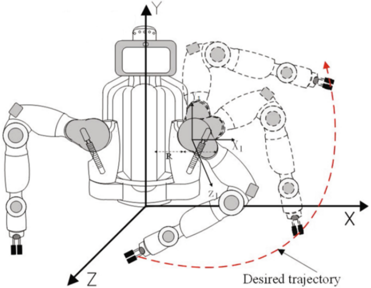

Figure 1: Tracking control

Time-Varying RISE and ARL Control Structure for Nonlinear Systems

- Combination of time-varying RISE[1] and ARL control structure for
- Robot manipulator: second-order CT nonlinear MIMO systems
- Sliding variable to achieve reduced-order system
- Transformed to autonomous affine system
- Comparison with RISE+ARL structure in [1]:
  - Clear exploration signal and initial conditions
  - Time-varying RISE implementation leads to better weight convergences

[1] Robust integral of the sign of the error

Disturbance Observer and ARL Control Structure for Nonlinear Systems

- Combination of disturbance observer, ARL-based control,
- Kinematic and feed-forward control structure for
- 3-DOF Surface vessel system: second-order CT nonlinear MIMO systems
- Transformed to autonomous affine system
- Comparison with previous works:
  - Simpler computation over [2] and [3]
  - Better tracking performance over [4]

Model of Robot Manipulator

The dynamic model of an n-link arm [1]:

$$M(\eta)\ddot{\eta} + C(\eta,\dot{\eta})\dot{\eta} + G(\eta) + F(\dot{\eta}) + d(t) = \tau \quad (1)$$

$\eta(t)$	joint variables
$M(\eta)$	inertia matrix
$C(\eta,\dot{\eta})$	Coriolis/centripetal matrix
$G(\eta)$, $F(\dot{\eta})$	gravity forces and friction
$d(t)$	disturbance vector
τ	control input

with several properties and necessary assumptions.

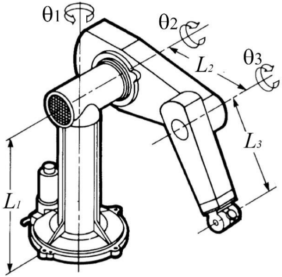

Figure 2: Illustration of a manipulator

Control Objective

Design a control structure for nonlinear system (1) in order to

- Track reference trajectory η_{ref} under disturbances d
- Minimize a predefined cost function $J(\cdot)$

Control Objective

Design a control structure for nonlinear system (1) in order to

- Track reference trajectory η_{ref} under disturbances d $\Rightarrow$ Time-varying RISE
- Minimize a predefined cost function $J(\cdot)$ $\Rightarrow$ On-policy AC

Firstly, with the joint error $e_1(t) = \eta_{ref} - \eta, \alpha_1 \in \mathbb{R}^{n\times n} > 0$, define a sliding variable

$$s(t) = \dot{e}_1 + \alpha_1 e_1 \tag{2}$$

Substitute into (1), the dynamics of $s(t)$ is obtained

$$M\dot{s} = -Cs - \tau + f + d \tag{3}$$

where $f = M(\ddot{\eta}_{ref} + \alpha_1 \dot{e}_1) + C(\dot{\eta}_{ref} + \alpha_1 e_1) + G + F$

With the system described in (3), design the control input

$$\tau = f + d - u = \varepsilon - u \tag{4}$$

ε – time-varying RISE estimation	$u(X)$ – On-policy AC-based control

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ε – time-varying RISE estimation $u(X)$ – On-policy AC-based control

The dynamics of $x = [e_1^T, s^T]^T$ is

$$\dot{x} = \begin{bmatrix} -\alpha_1 e_1 + s \\ -M(\eta_{ref} - e_1)^{-1} C(\eta_{ref} - e_1, \dot{\eta}_{ref} + \alpha_1 e_1 - s)s \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{n \times n} \\ M^{-1} \end{bmatrix} u \tag{5}$$

To obtain **autonomous affine system**, assume that $\dot{\eta}_{ref}(t) = f_{ref}(\eta_{ref})$, then

$$\dot{X} = A(X) + B(X)u \tag{6}$$

with $X = [x^T, \eta_{ref}^T, \dot{\eta}_{ref}^T]^T$ and the matrices:

$$A(X) = \begin{bmatrix} -\alpha_1 e_1 + s \\ -M(\cdot)^{-1} C(\cdot) s \\ f_{ref}(\eta_{ref}) \\ \dot{f}_{ref}(\eta_{ref}) \end{bmatrix}, \quad B(X) = \begin{bmatrix} \mathbf{0} \\ M^{-1} \\ \mathbf{0} \end{bmatrix} \tag{7}$$

The infinite horizon cost function to be minimized:

$$J = \int_0^\infty \left(\frac{1}{2} X^T Q_T X + \frac{1}{2} u^T R u\right) dt \tag{8}$$

On-Policy Actor-Critic Architecture

The infinite horizon cost function to be minimized:

$$J = \int_0^\infty (\frac{1}{2}X^T Q_T X + \frac{1}{2}u^T R u)dt \tag{8}$$

Hard-to-solve HJB equation with the optimal value function $V^*(X)$ and the optimal feedback controller $u^*(X) \rightarrow$ **approximated using a NN**

$$V^*(X) = W^T \psi(X) + \varepsilon_v(X), \tag{9}$$

$$u^*(X) = -\frac{1}{2}R^{-1}B^T(X)\Big((\frac{\partial \psi}{\partial X})^T W + (\frac{\partial \varepsilon_v(X)}{\partial X})^T\Big) \tag{10}$$

On-Policy Actor-Critic Architecture

Using two separate NNs for easier weight update and stability analysis [5]

$$\hat{V}(X) = \hat{W}_c^T \psi(X), \quad \hat{u}(X) = -\frac{1}{2} R^{-1} B^T(X) \left(\frac{\partial \psi}{\partial X}\right)^T \hat{W}_a \tag{11}$$

Actor and critic approximators are tuned simultaneously to minimize Bellman error δ_{hjb}. The least-squares update of critic weights is

$$\dot{\hat{W}}_c = -k_c \lambda \frac{\sigma}{1 + \upsilon \sigma^T \lambda \sigma} \delta_{hjb}, \quad \dot{\lambda} = -k_c \lambda \frac{\lambda \sigma^T}{1 + \upsilon \sigma^T \psi \sigma} \lambda \tag{12}$$

The actor adaptation law is based on gradient descent method

$$\dot{\hat{W}}_a = -\frac{k_{a1}}{\sqrt{1 + \sigma^T \sigma}} \frac{\partial \psi}{\partial X} B R^{-1} B^T \frac{\partial \psi^T}{\partial X} (\hat{W}_a - \hat{W}_c) \delta_{hjb} - k_{a2} (\hat{W}_a - \hat{W}_c) \tag{13}$$

ε is the estimation of $f + d$, based on the time-varying RISE [6].

$$\varepsilon(t) = (K_s(\cdot) + 1)s(t) - (K_s(t_0) + 1)s(0) + \rho(t) \tag{14}$$

$$\dot{\rho} = (k_{s0} + 1)\alpha(\cdot)s(t) + \beta\,\mathrm{sgn}(s(t)) \tag{15}$$

with $K_s(\cdot)$ and $\alpha(\cdot)$ are two nonlinear feedback functions designed as

$$K_s(\cdot) \equiv K_s(s, \gamma_1, \delta_1) = \begin{cases} k_{s0}|s|^{\gamma_1 - 1}, |s| > \delta_1 \\ k_{s0}{\delta_1}^{\gamma_1 - 1}, |s| \leq \delta_1 \end{cases} \tag{16}$$

$$\alpha(\cdot) \equiv \alpha(s, \gamma_2, \delta_2) = \begin{cases} \alpha_0|\int s|^{\gamma_2 - 1}, |\int s| > \delta_2 \\ \alpha_0{\delta_2}^{\gamma_2 - 1}, |\int s| \leq \delta_2 \end{cases} \tag{17}$$

where $k_{s0}, \alpha_0, \gamma_1, \delta_1, \gamma_2, \delta_2$ are positive parameters which need designing carefully.

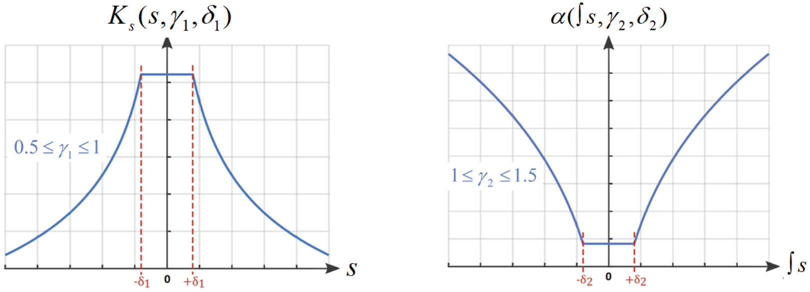
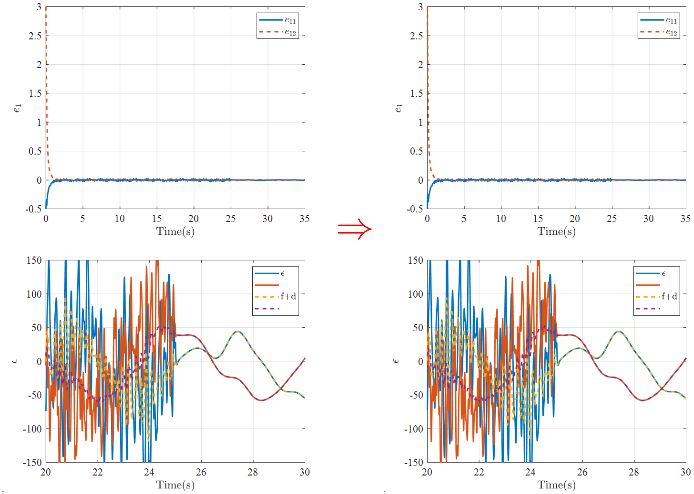


Figure 3: Functions of the control gains with respect to their arguments



- RMSE: 0.2073 and 0.2072
- Control reduces 0.52%
- Similar estimation

Figure 4: Tracking errors and estimation of the two controllers

Simulation Results

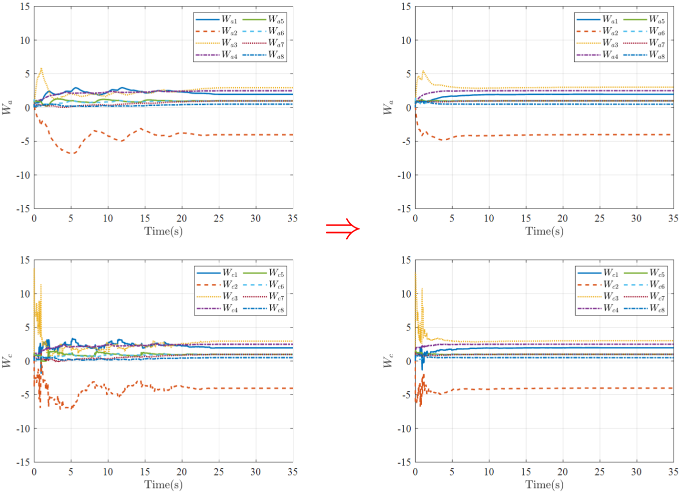


Figure 5: Weight convergences

- Both show great convergences
- Convergence error reduces 41.63%
- Faster and smoother

$\eta = [x, y, \psi]^T$ denotes the 3-DOF position
$\nu = [u, v, r]^T$ denotes the corresponding velocities
The model of SV system [7]:

$$\begin{cases} \dot{\eta} = R(\eta)\nu(t) \\ M\dot{\nu} = \tau + \Delta(\eta, \nu) - f(\eta, \nu) \end{cases} \quad (18)$$

with dynamics $f(\eta, \nu)$ is modeled by

$$f(\eta, \nu) = C(\nu)\nu + D(\nu)\nu + g(\eta, \nu) \quad (19)$$

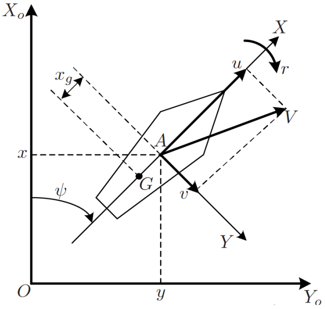


Figure 6: Earth-fixed and body-fixed coordinate frames of an SV

Control Objective

Design a control structure for nonlinear system (18) in order to

- Track reference trajectory η_d
- Robust with disturbances Δ
- Minimize a predefined cost function $J(\cdot)$

Control Objective

Design a control structure for nonlinear system (18) in order to

- Track reference trajectory $\eta_d \Rightarrow$ Kinematic and feed-forward control
- Robust with disturbances $\Delta \Rightarrow$ Disturbance observer
- Minimize a predefined cost function $J(\cdot) \Rightarrow$ On-policy AC

Kinematic and Feed-Forward Control Structure

With the trajectory tracking error $e_\eta = \eta - \eta_d$, design the kinematic control law

$$\nu_d = R^{-1}(\eta)(\dot{\eta}_d - \beta_\eta e_\eta) \tag{20}$$

Kinematic and Feed-Forward Control Structure

With the trajectory tracking error $e_\eta = \eta - \eta_d$, design the kinematic control law

$$\nu_d = R^{-1}(\eta)(\dot{\eta}_d - \beta_\eta e_\eta) \tag{20}$$

With the body-fixed velocity error: $e_\nu = \nu - \nu_d$, a feed-forward term τ_{ff} is added

$$\tau = u + \tau_{ff}, \quad \tau_{ff} = M\dot{\nu}_d + f(\eta_d, \nu_d) \tag{21}$$

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$$\tau = u + \tau_{ff}, \quad \tau_{ff} = M\dot{\nu}_d + f(\eta_d, \nu_d) \tag{21}$$

Let $X = [e_\nu^T \quad e_\eta^T \quad \eta_d^T]^T$ is an augmented state for the dynamic subsystem of SV. The **autonomous system** can be represented concisely as

$$\dot{X} = F(X) + G_u(X)u + G_d(X)\Delta \tag{22}$$

Kinematic and Feed-Forward Control Structure

The control input is designed as

$$u = d(X)\hat{\Delta} + u_r(X) \tag{23}$$

$d(X)\hat{\Delta}$ – disturbance compensator $u_r(X)$ – RL-based control for nominal system

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$d(X)\hat{\Delta}$ – disturbance compensator $u_r(X)$ – RL-based control for nominal system

According to [8], with $\dot{\Delta} \simeq 0$, a disturbance observer is constructed

$$\begin{cases} \hat{\Delta} = y + P(X) \\ \dot{y} = -\frac{\partial P(X)}{\partial X}(G_u(X)u + G_d(X)y + G_d(X)P(X) + F(X)) \end{cases} \tag{24}$$

If $\frac{\partial P(X)}{\partial X} G_d(X)$ is positive definite, $\tilde{\Delta}$ is exponentially stable, as $t \to \infty$.

The disturbance compensation gain is

$$d(X) = -G_u^{+} G_d \tag{25}$$

Disturbance Observer and ARL Control Structure

Simulation Results

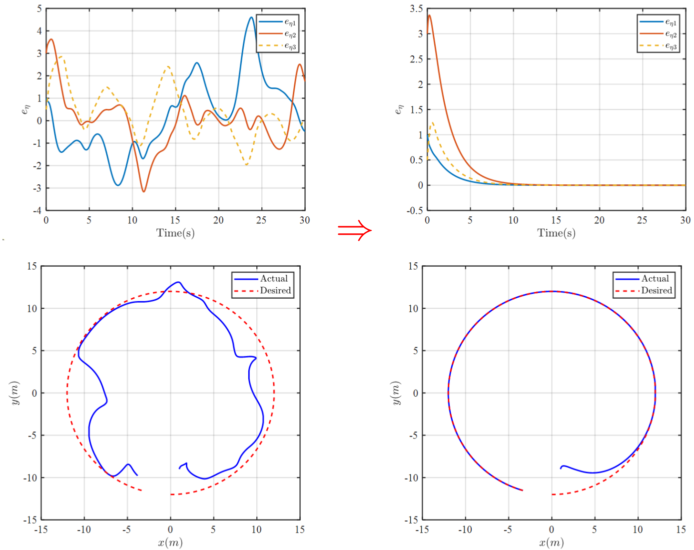

Figure 7: Tracking errors and trajectories of the two controllers

- RMSE reduces 66%

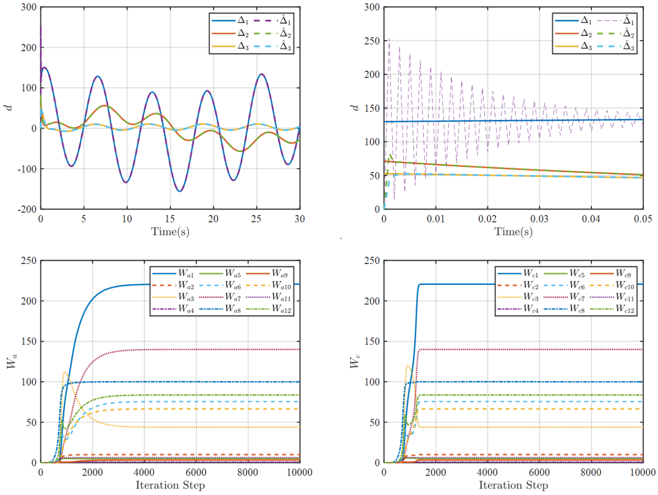

Figure 8: Estimation error and weight convergence

- Fast and precise estimation
- AC convergence

Conclusion

Two problems are accomplished satisfying:

- ✓ Great trajectory tracking performance
- ✓ Robustness to disturbance
- ✓ Minimization of predefined cost function

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Successful implementation of frameworks:

- ✓ System modeling
- ✓ On-policy actor-critic architecture (ARL/ADP)
- ✓ Sliding variable, kinematic and feed-forward control
- ✓ Time-varying RISE, and disturbance observer...

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Limitations:

- Requirement of drift matrix and input gain matrix
- Lack of some explicit proof of stability and optimality

Future Work

- A number of intriguing unresolved challenges
- Partially/completely model-free RL approaches
- Explicit optimality and stability demonstration
- Differential games and multi-agent systems

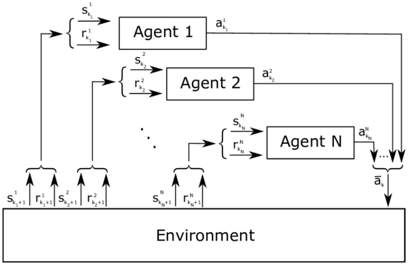

Figure 9: RL of multi-agent system

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Thank you!